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ON THE DISTRIBUTION OF SURFACE HEAT FLOWS AND THE SECOND ORDER VARIATIONS IN THE EXTERNAL GRAVITATIONAL FIELD

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1. Introduction

Knowledge about the earth's external gravitational field has increased significantly in recent years, mainly because of observations on close satellite orbits but also because of the extended coverage of the surface gravity measurements. Analyses of secular and long-period perturbations of satellite orbits have yielded accurate determinations of the zonal harmonics up to the 9th order; a summary of the results was given by Kozai (1962). Meanwhile, improved determinations of the tesseral harmonics based on the analysis of different types of short-period perturbations of satellite orbits are being made (Kaula, 1963; Izsak, 1963). The results are compatible with some recent analyses of the surface gravity data (Uotila, 1962). The principal features of the second-order variations in the external gravitational field as indicated by these studies can be summarized by plotting the undulations of the geoid above a best-fitting ellipsoid. These undulations, called the geoid-heights, show little if any correlation with the surface topographical features.

The simplest explanation of the second-order undulations of the geoid is the assumption that density anomalies of small magnitude exist somewhere in the interior of the earth. The density anomaly may be due to inhomogeneities in the composition or in some other properties of the earth's mantle. A finite strength of the mantle or a system of convection currents has been assumed, to support the different loads associated with such density anomalies. The finite strength hypothesis has been discussed by O'Keefe (1959), McDonald (1962), and Kaula (1963a), while Licht (1960) has used a model of classical, viscous fluid in explaining the third order zonal harmonic. Direct verification of these hypotheses is not yet possible. In this report I attempt to find an explanation for the second order terms in the gravity field, based on the distribution of heat flows over the earth's surface.

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2. Geoid heights

The geoid is defined as the gravitational equipotential surface of the earth, which coincides with the mean sea-level surface over the oceans. The geoid is approximately an ellipsoid that bulges at the equator and is slightly flattened at the poles, partly because of the rotation of the earth and partly because of the shape of the solid earth.

The shape of the geoid can be computed from the free-air gravity anomalies over a complete level surface; it can also be computed from the earth's external gravitational potential. The geoid is not exactly ellipsoidal. The departures from the best-fitting ellipsoid can in general be classified into two types; one has small amplitudes (a few tens of meters) but keeps the same sign over distances comparable to the length of the earth's radius, while the other changes signs from place to place irregularly and has various amplitudes depending on the immediate local geological structures. The analyses of local departures need a standard for comparison, and the best standard would be a function that best fits the geoid as a whole; that is, one based on the low-order spherical harmonics. I. G. Izsak of the Smithsonian Astrophysical Observatory has made calculations of the numerical values of the coefficients of the low-order spherical harmonics, by analyzing the satellite orbits. The geoid heights shown in figure 1 are plotted on a reference ellipsoid with flattening 1/298.27, and are based on the numerical results given in table 1.

3. Heat flow

I shall restrict the discussion to the heat flow to the surface of the earth by conduction through the solid crust. The heat flow at a particular location is the product of the two quantities: the conductivity of the rock and the temperature gradient. The top few tens of meters of the continental crust are subjected to seasonal variations of temperature. Below this thin layer of crust, temperature increases steadily with depth. The temperature gradient below this thin layer can be measured in bore holes a few hundred meters deep, by use of a probe that contains a thermally sensitive resistor. The probe is let down in the bore hole to certain depths where the temperatures are measured. Most of the bore holes in which the temperature gradients were measured were drilled for mining purposes and therefore were located at areas where the geology is complex. However, the Geophysics Department of Harvard University is now carrying out a project to drill holes at locations especially suitable for heat flow measurements. Heat flows have also been measured in the oceanic areas. The temperature at the bottom of the ocean is steady. Temperature gradients are measured by plunging the probe into the bottom material and letting it remain there until the temperature approaches equilibrium. Samples of the bottom material are collected by the corer and the thermal conductivities of the samples are measured on the ship.

In recent years, heat flow measurements have been made extensively over the earth. The distribution of the measurements, however, is uneven. For example, no data are available from the Asian or South American continents, the Indian Ocean, a large part of the European continent, or areas near the South and North Poles. Few data are available from the Australian and the African continents, the central part of the Pacific Ocean, or the South Atlantic. Because of these unsatisfactory, uneven distributions, it seems too early at this moment to make detailed analyses of heat flow data. However, it is interesting to plot the data available and to see the general pattern of distribution over the areas where measurements have been made. The writer has about 700 heat flow measurements at hand, most of them from Lee (1963). Among these, only 668 have definite longitudes and latitudes. About 85 percent of the measurements come from the oceanic area near the American and European continents; the rest are concentrated mostly in Japan, North America, the British Isles, Hungary, and Germany. I have divided the 668 measurements into five groups with boundaries at 1.0, 1.5, 2.5 and 3.5 $\mu cal/cm^2 sec$, and have plotted the results on a world map.

The arithmetical mean of these data is found (Lee, 1963) to be 1.5 $\mu cal/cm^2$ sec. By averaging the data on the basis of $10^o \times 10^o$ squares, we find four classes of heat flow areas: low (average value less than 1.0), medium (average value between 1.0 and 1.5), high (average value between 1.5 and 2.5), and very high heat flow (average value above 2.5 $\mu cal/cm^2 sec$). The result is shown in figure 2. The west coast of the United States and the east Pacific rise stand out as regions of very high heat-flow, as do the mid-Atlantic ridge, the Alps, and the islands of Japan.

In figure 3 the average values of heat flow are taken for $20^o \times 20^o$ squares, and the areas are classified into two groups: those with the high and those with the low heat flows, with the dividing line at 1.5 $\mu cal/cm^2$ sec (squares with only one measurement are not taken into account). This figure clearly shows that the distribution of the high and the low heat flows has no correlation with the distribution of the continents and oceans, in spite of the fact that granite, which comprises most of the continental crust, contains much larger amounts of the radioactive elements than does basalt, which comprises the oceanic crust. Certain factors controlling the distribution of heat flows must operate below the earth's crust.

4. Possible correlation between gravity and heat flow

In figure 1 the $20^o \times 20^o$ squares, corresponding to those with heat flow measurements in figure 3, are classified as squares of high and of low gravity according to whether the geoid height of the square is positive or negative. When a given square has both positive and negative geoid heights, the average value is taken as representative.

Comparing figure 1 and figure 3, we find some correspondence between the high heat flow squares and the low gravity squares, and between the low heat flow squares and the high gravity squares. This correspondence is more clear if we disregard the data obtained from areas near active volcanoes (for example, Japan and the mid-Atlantic ridge), whose very high heat flows are likely to be related to the volcanic activity. The correlation coefficient is found to be 0.5.

The close correspondence between the highs and lows of the squares in figures 1 and 3 suggests that some common factor or factors control both sets of data and influence the distributions. Hotter regions with lower density beneath the crust must exist under areas of high heat flow, and vice versa. An interpretation for both sets of data (gravity and heat flow), based on the convection current hypothesis, is given below.

The theory of convection currents is complicated. The initiation of a convection cell depends upon the combination of many factors: the temperature gradient, the coefficient of thermal expansion of the mantle material, the magnitude of the associated shear stresses, the hydrostatic pressure, the strength of the material, etc. Since our knowledge about these properties of the interior is extremely limited we have to restrict ourselves to a simple model. From the hydrostatic phenomena of the continents and oceans we can assume that the mantle behaves as a fluid under shear stresses of small magnitude with periods of millions of years. Suppose that a part of the mantle near the core-mantle boundary, for example, is heated relatively more than the surrounding material; the heated material tends to expand and thus acquires a lower density; it then tends to float upwards where the pressure is lower. Heat is then brought up. A hot column with relatively lower density than its surroundings may thus come into existence. Near the top of the mantle, where a finite strength with long period possibly exists, further upward floating is no longer possible, and the hot material tends to overflow to the surrounding regions. Near the bottom of the hot column the surrounding cooler material may encroach into the hot column and be heated up there. A descending current will then occur in the cold columns and the process thus becomes cyclic (fig. 4). Thus an area of high surface heat flow should have a lower gravity, and vice versa. The correlation of figure 1 with figure 3 is thus what is to be expected from the above theory.

Licht (1960) suggested a model of convection currents in a fluid mantle to explain the existence of the third-order harmonic in the gravity field. Three types of perturbations in the mantle were taken into consideration: the density of the mantle, the shape of the crust-mantle boundary, and that of the core-mantle boundary. The crust-mantle boundary was supposed to be pushed upwards through a distance over a hundred meters by the ascending currents, and the resulting gravity field over the ascending currents was thought to be higher than that over the descending currents. However, the existence of the continents and oceans, especially the mountain ridges and the oceanic deeps, indicates that the uppermost 50 km of the earth has a long period strength at least of the order of 1500 bars (Jeffreys, 1958, p. 210). It is therefore doubtful that the crust would have yielded under the stresses associated with the slowly flowing currents (a few centimeters per year, according to Licht) to give extra mass over the ascending currents and deficient mass over the descending currents. Furthermore, according to Licht's theory higher gravity should be correlated with high surface heat flow, and vice versa, correlations that apparently do not occur.

McDonald (1962), on the other hand, suggested that the second-order terms may be due to convection currents in the mantle such that denser material is supported by ascending currents and thus produces higher gravity over it. However, since it is the lighter material that tends to float, the ascending current should be composed of material lighter, not heavier, than the surrounding material. Furthermore, McDonald's theory implies as does Licht's, that areas with high surface heat flow should at the same time be areas with higher gravity, etc., which apparently is not true.

Mathematical development of the convection current theory is complicated unless the problem is greatly simplfied by assuming uniform or simply varying viscosity and conductivity of the mantle material, a simple model of temperature gradient, definite size of the convection cells, classical behavior of the mantle material, etc. Such work is not attempted here. The writer estimates, however, that a uniform distribution of density anomalies of the order of 10^{-3} gm/cm^3 through the top 300 km of the earth's mantle could account for the second-order terms in the gravity field. Estimation of the temperature differences associated with these density anomalies depends, of course, on knowledge of the thermal expansion coefficients of the mantle material, which is of the order of 10^{-5} deg^{-1}. The corresponding temperature difference is of the order of 10 degrees in centigrade.

5. Conclusion

The material under the depressed geoid may be lighter and hotter, the one being related to the other. Motions are not necessarily indicated. However, these correlated phenomena may be explaind by thermal convections in the mantle such that ascending currents composed of lighter material bring up heat to the top of the mantle, and that high surface heat flow appears over the ascending currents, while the gravity is lower than its surroundings.

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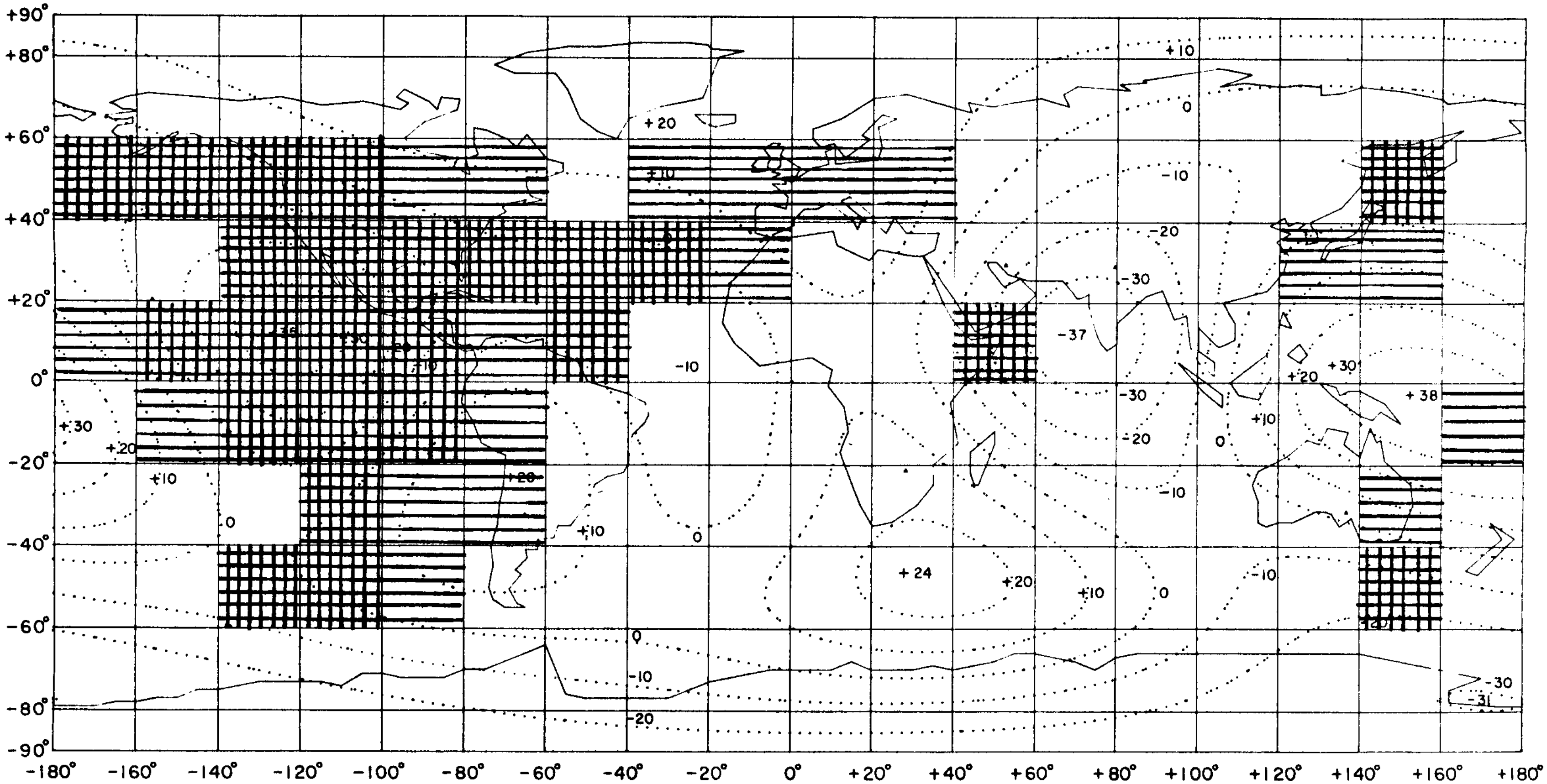

Figure 1.-- Geoid height in meters above a reference ellipsoid with flattening 1/298.27
positive geoid height, negative geoid height. (From Izsak, 1963)

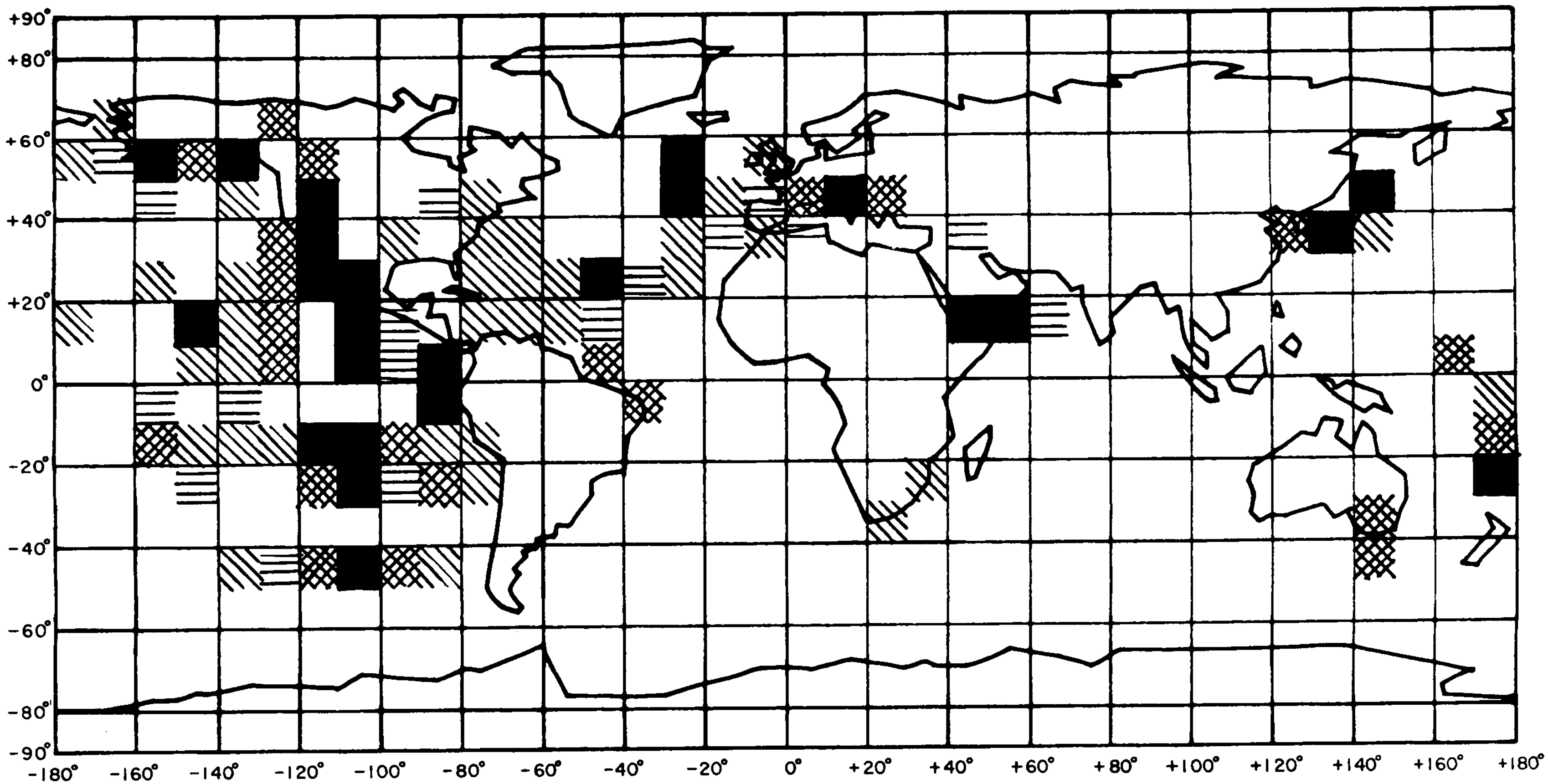

Figure 2.-- Average values of heat flow for the 10° x 10° squares (in μ cal/cm^2 sec) 2.50 and larger, 1.50-2.49, 1.00-1.49, 0-0.99.

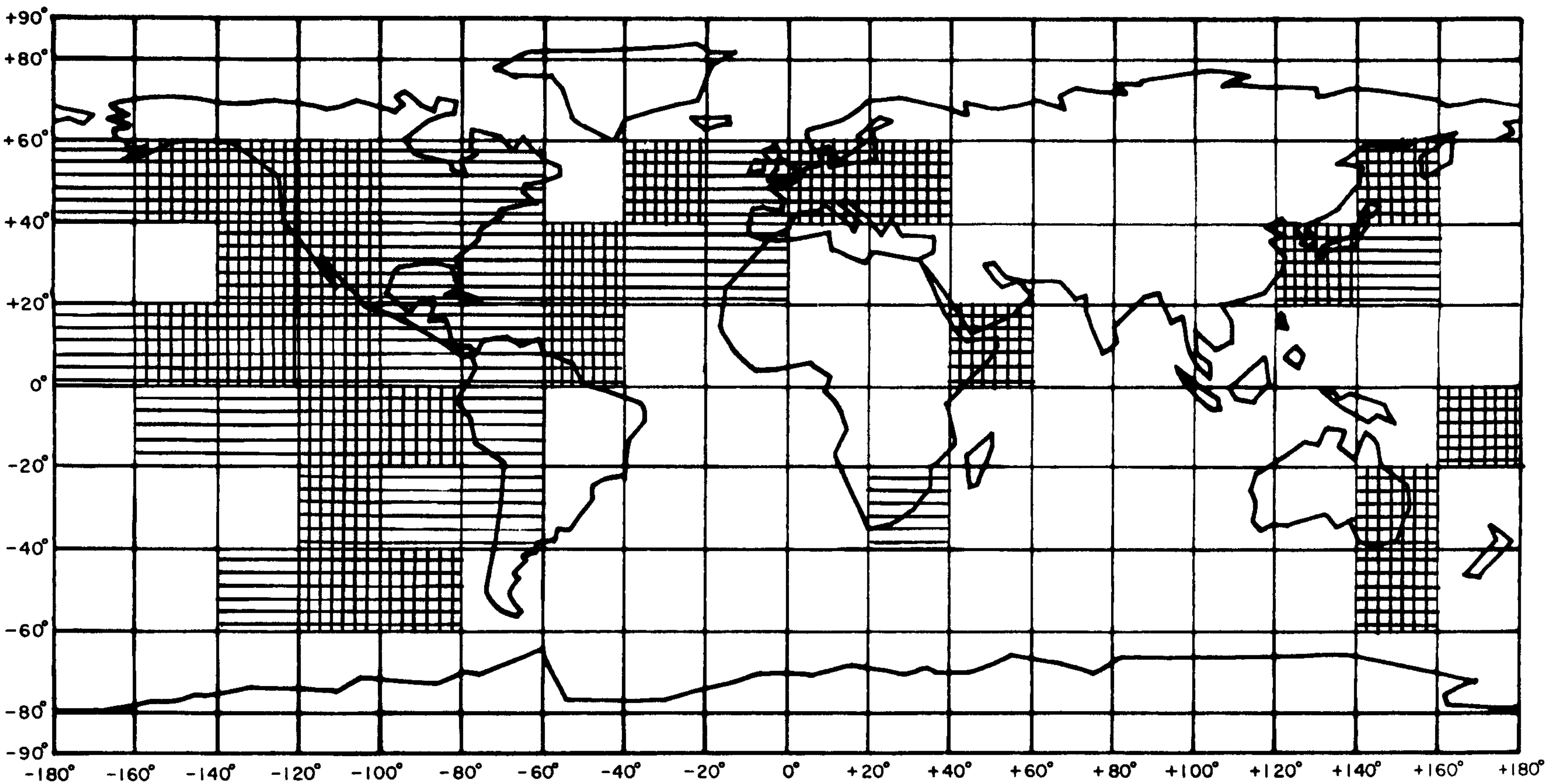

Figure 3.-- Average values of heat flow for the 20° x 20° squares.
High heat flow, with average values larger than 1.5 $\mu cal/cm^2$ sec.
Low heat flow, with average values smaller then 1.5 $\mu cal/cm^2$ sec.

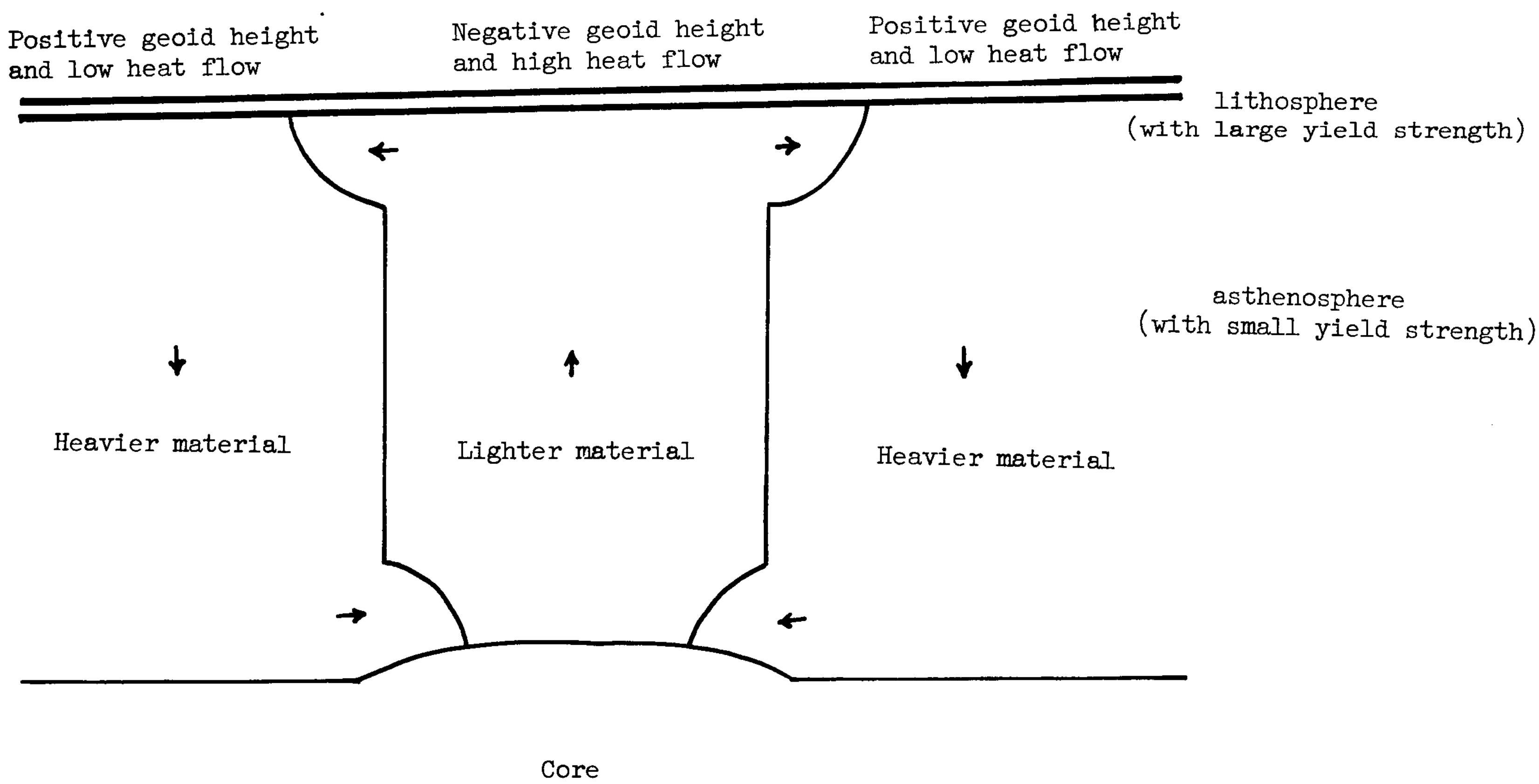

Figure 4.-- Schematic representation of a convection cell and its relation with the surface expressions.

Table 1.--Numerical values of C_{nm} and S_{nm} in the external gravitational potential U

$$U = \frac{GM}{r}\left\{1 + \sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \left[C_{nm} \cos m\lambda + S_{nm} \sin m\lambda\right] P_{nm}(\sin\beta)\right\}$$

Zonal harmonics
(from Kozai, 1962)

$$\begin{aligned}
C_{20} &= -1082.48 \times 10^{-6} \\
C_{30} &= 2.562 \times 10^{-6} \\
C_{40} &= 1.84 \times 10^{-6} \\
C_{50} &= 6.4 \times 10^{-8} \\
C_{60} &= -3.9 \times 10^{-7} \\
C_{70} &= 4.7 \times 10^{-7} \\
C_{80} &= 2 \times 10^{-8} \\
C_{90} &= -1.17 \times 10^{-7}
\end{aligned}$$

Tesseral harmonics
(from Izsak, 1963)

$C_{22} = 9.68 \times 10^{-7}$	$C_{41} = -2.88 \times 10^{-7}$
$S_{22} = -4.00 \times 10^{-7}$	$S_{41} = -3.21 \times 10^{-7}$
$C_{31} = 1.12 \times 10^{-6}$	$S_{42} = 3.51 \times 10^{-8}$
$S_{31} = 6.16 \times 10^{-8}$	$S_{42} = 1.23 \times 10^{-7}$
$C_{32} = 9.12 \times 10^{-8}$	$C_{43} = 2.15 \times 10^{-8}$
$S_{32} = -1.83 \times 10^{-7}$	$S_{43} = 1.48 \times 10^{-8}$
$C_{33} = 7.17 \times 10^{-8}$	$C_{44} = 9.72 \times 10^{-9}$
$S_{33} = 1.24 \times 10^{-7}$	$S_{44} = 1.63 \times 10^{-8}$